# Written Exam at the Department of Economics winter 2020-21

## Advanced Economics of the Environment and Climate Change

Final Exam

16 January 2021

(3½-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 5 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

#### Be careful not to cheat at exams!

You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

# Exercise 1. Environmental regulation and inequality (indicative weight: 3/4)

Consider a small market economy with N households indexed i = 1, 2, ..., N. Households supply labor inelastically. Units are chosen such that each household supplies one unit of labor.

The households differ in their productivity. The effective labor supplied by household *i* is given by the labor supply (which equals one) multiplied by the productivity measure  $\phi_i$ . These productivity measures are normalized such that:

$$\sum_{i=1}^{N} \phi_i = 1$$

Accordingly, the effective labor supply of the entire economy equals one.

Household *i*'s wage income,  $I_i$ , is given by:

$$I_i = \phi_i w (1 - \tau_w^0), \tag{1}$$

where w is an exogenous (average) wage rate, and  $\tau_w^0$  is the pre-reform wage tax.

The utility of household i is given by:

$$U = C_i^{\alpha} (D_i - D_0)^{\beta} - \kappa D + \lambda \overline{G},$$
  
$$0 < \alpha < 1, \quad 0 < \beta < 1, \quad \alpha + \beta = 1, \quad \kappa \ge 0, \quad \lambda \ge 0,$$

where  $C_i$  measures consumption of non-polluting goods,  $D_i$  measures the consumption of polluting goods,  $D_0 > 0$  is the subsistence level (minimum-consumption requirement) for polluting consumption, D is aggregate pollution emission (from domestic territory), and  $\bar{G}$ is a public good provided by the government. In the pre-reform scenario there is no public good:  $\bar{G} = 0$ .

The budget constraint of household i is given by:

$$C_i p_C + D_i p_D (1+\tau) = L_i + I_i,$$

where  $p_C$  and  $p_D$  are exogenous prices on non-polluting and polluting goods, respectively,  $\tau$  is the ad valorem pollution tax, and  $L_i$  is a lump-sum transfer. In the pre-reform scenario

there is no pollution tax and no lump-sum transfer:  $\tau = L_i = 0$ .

Each household maximizes utility, U, subject to the budget constraint taking all prices, policies, and aggregate variables as given. All households have an income that allow them to purchase more than just the subsistence level of polluting goods.

The government's budget constraint is given by:

$$G + \bar{G} + \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \tau p_D D_i + \sum_{i=1}^{N} \tau_w^0 \phi_i w$$

where G is a fixed spending requirement of the government. The equation states that the government's spendings (left-hand side) equals the government's tax revenue (right-hand side).

The consumption of polluting goods results in pollution emissions. Units are chosen such that consuming one unit of the polluting good results in one unit of pollution emission. Hence aggregate pollution emission equals the total consumption of polluting goods:

$$D = \sum_{i=1}^{N} D_i.$$

Finally, define *actual disposable income* as:

$$\bar{I}_i \equiv \phi_i w (1 - \tau_w^0) + L_i - D_0 p_D (1 + \tau).$$

The actual disposable income is the disposable income left after purchasing the subsistence level of polluting goods.

#### Question 1.1

Show that solving the problem of household i implies that:

$$C_i = \frac{\alpha}{p_C} \bar{I}_i$$
, and  $D_i = \frac{\beta}{p_D(1+\tau)} \bar{I}_i + D_0$ .

#### Question 1.2

How does the ratio between expenditures on polluting and non-polluting goods depend on income? Discuss how this prediction is aligned with empirical evidence.

From here we will interpret pollution as air pollution.

#### Question 1.3

Discuss for which air pollutants it is reasonable to assume  $\kappa = 0$  and  $\kappa > 0$ , respectively. Hint: the economy is small.

Define the indirect utility function:

$$V_i(\cdot) = U\left(C_i(\tau_w^0, L_i, \tau, \phi_i), D_i(\tau_w^0, L_i, \tau, \phi_i)\right) - \kappa D(\cdot) + \lambda \bar{G}(\cdot),$$

where  $V_i$ , D and  $\overline{G}$  are functions of  $\tau_w^0$ ,  $\phi_1$ , ...,  $\phi_N$ ,  $L_1$ , ...,  $L_N$ , and  $\tau$ .

Consider the case  $\kappa = 0$ . The government introduces an environmental tax reform. The reform implies that pollution emissions are taxed,  $\tau > 0$ , and that the entire tax revenue from the reform is transferred back to the households via lump-sum transfers:

$$\sum_{i=1}^{N} L_i = p_D \tau \sum_{i=1}^{N} D_i.$$

All households receive the same lump-sum transfer which amounts to:

$$L = p_D \tau \frac{D}{N}.$$

#### Question 1.4

Show that the reform is progressive in the sense that  $V_i(\cdot)/V_j(\cdot)$  increases compared to the pre-reform scenario, when  $\phi_i < \phi_j$ . Comment on the interpretation of this measure of relative welfare. *Hint: note that*  $\kappa = 0$  and  $\bar{G} = 0$ .

#### Question 1.5

How does the reform affect income inequality between household i and j (still assuming  $\phi_i < \phi_j$ )? Use the relative income between household i and j before and after the reform to answer the question. Briefly explain the intuition.

Through the remaining part of this exercise, consider the case  $\kappa > 0$  and  $\lambda > 0$ , and the following environmental tax reform. The government introduces a tax on pollution emission:  $\tau > 0$ . The tax revenue is used for two purposes.

Firstly, the government provides a lump-sum transfer to all households that covers additional expenditures on the subsistence level of polluting consumption imposed by the pollution emission tax:

$$L_i = L = p_D \tau D_0.$$

The total cost of this transfer is:

$$\sum_{i=1}^{N} L = NL = Np_D \tau D_0.$$

Secondly, the government uses the remaining revenue to finance a public good. This implies that:

$$\bar{G} = \underbrace{p_D \tau \sum_{i=1}^N D_i}_{\text{Total tax revenue}} - \underbrace{N p_D \tau D_0}_{\text{Cost of transfer scheme}} = p_D \tau \sum_{i=1}^N (D_i - D_0).$$

One can show that aggregate pollution emission after the reform is given by:

$$D = \frac{\beta w(1 - \tau_w^0)}{p_D(1 + \tau)} + ND_0 \left(1 - \frac{\beta}{1 + \tau}\right).$$

You can take this result as given.

# Question 1.6

Show that aggregate pollution emission is declining in the pollution tax  $\tau$ . Explain the role of the subsistence level of polluting consumption.

The optimal emission tax for household i given the transfer scheme, denoted  $\tau_i^*$ , can be derived from the problem:

$$\max_{\tau} \left(\frac{\alpha}{p_C}\right)^{\alpha} \left(\frac{\beta}{p_D(1-\tau)}\right)^{\beta} \bar{I}_i - \kappa D + \lambda \bar{G}.$$

It can be shown that  $\tau^*_i$  can be expressed as:

$$\tau_i^* = \left[\frac{\beta \left(\lambda + \kappa/p_D\right) \sum_{i=1}^N \bar{I}_i}{\left(\frac{\alpha}{p_C}\right)^{\alpha} \left(\frac{\beta}{p_D}\right)^{\beta} \bar{I}_i}\right]^{\frac{1}{1-\beta}} - 1.$$

You can take this expression as given.

#### Question 1.7

Do the households agree on the optimal emission tax level given the transfer scheme? Explain the intuition.

#### Question 1.8

Evaluate the usefulness of this model in terms of analyzing equity (or inequality) issues related to environmental tax reforms. Your evaluation should focus on mechanisms that are absent in the model.

# Exercise 2: The connection between the Hotelling rule and the Green Paradox hypothesis

# (indicative weight: 1/4)

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful)

## Question 2.1

Briefly explain the Hotelling rule.

#### Question 2.2

Discuss how the Hotelling rule is connected to the Green Paradox hypothesis. Discuss how the empirical validity of the Hotelling rule affects the validity of the Green Paradox hypothesis. Start your discussion with a brief introduction of the Green Paradox hypothesis.